Section Title : One Mark Questions
Total Questions: 10
Max Marks : 1
-ve Marks :0.33

Question No: 1

Analysis

If the matrix A of order 5×5 has two non-zero eigen values then the rank

of a matrix A

)	3
•)

(C) 4 (D) 5

Solution :

.:. Option (a) is correct.	Ouestion No: 2
values	
The rank of the matrix A $_{n \times n}$ is equal to the number of non-zero eigen	

Analysis

Which of the following is the 3rd order and 2nd degree differential equation?

(A)
$$\left(\frac{d^3y}{dx^3}\right)^4 + \frac{dy}{dx} + \sin(y) = 0$$
 (B) $\frac{\partial^2 u}{\partial x^2} + \left(\frac{\partial^3 U}{\partial x \partial y^2}\right)^2 + u = 0$

(C)
$$\left(\frac{\partial^3 u}{\partial x^3}\right)^2 + \left(\frac{\partial^3 u}{\partial y^3}\right)^5 + \frac{du}{dy} = 4$$
 (D) $\frac{dy^4}{dx^4} + \frac{dy}{dx} + \cos(x) = 0$

Not Attempted -- Correct Answer

Not Attempted -- Correct Answer : A

: B

Solution :

By the definition of order and degree of the differential equation, option (b) is correct.

Question No: 3

Analysis

If $f(x, y) = x^3 + y^3 - 6xy$ then the point (2, 2) is

(A) a point of minima

(C) a saddle point

(D) both (A) & (B)

Not Attempted -- Correct Answer : A

Solution :

Given
$$f(x, y) = x^3 + y^3 - 6xy$$

 $\Rightarrow P = f_x = 3x^2 - 6y, q = f_y = 3y^2 - 6x$
And $r = f_{xx} = 6x, S = f_{xy} = -6, t = f_{yy} = 6y$
At (2, 2): $r = 12, s = -6, t = 12$
Now $rt - s^2 = (12) (12) - (-6)^2 = 144 - 36 > 0$
But $r = 12 > 0$
 \therefore (2, 2) is a point of minima

If
$$u = log\left(\frac{x^4 + y^4}{x + y}\right)$$
 then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$ ______.

Not Attempted -- Correct Answer : A

Solution :

The given function u(x, y) is not a homogeneous function but $f(u) = e^{u} = \left(\frac{x^{4} + y^{4}}{x + y}\right)$ is a homogeneous function with degree n = 4 - 1 = 3By Euler's deduction formula, we have $xu_{x} + yu_{y} = n \frac{f(u)}{f^{1}(u)}$ $\therefore xu_{x} + yu_{y} = 3 \frac{e^{u}}{e^{u}} = 3$ Question No: 5

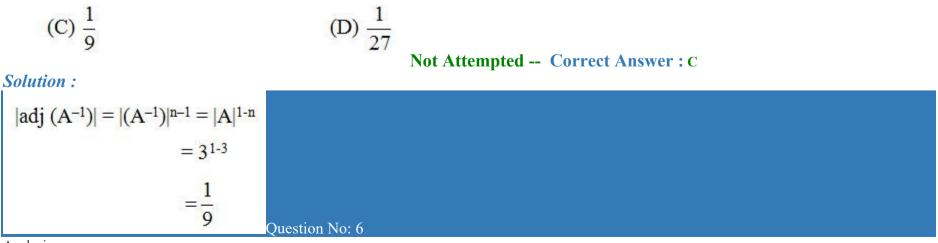
Question No: 4

Analysis

Let A be a non-singular matrix of order 3×3 .

If det(A) = 3 then $det(adjA^{-1})$ is

(A)
$$\frac{1}{3}$$
 (B) $\frac{1}{6}$



Analysis

The curl of gradient of scalar function

$$\phi = x^2 y^2 z^4 is$$

(A) –1 (B) 1

(C) 0 (D) 2

Not Attempted -- Correct Answer : C

Solution :

Curl (grad ϕ) = 0 (Vector Identity) Question No: 7

Analysis

The minimum value taken by the function

$$f(x) = \left[\frac{|x|}{1+|x|}\right] - 1 \text{ is } _$$

Not Attempted -- Correct Answer : -1 & Valid Answer Range :-1,-1 Solution :

$$f(x) = \frac{|x| - (1 + |x|)}{(1 + |x|)} = \frac{-1}{1 + |x|}$$

$$f'(x) = \frac{1}{(1 + |x|)^2} \frac{|x|}{x}$$

$$f'(x) = 0 \implies x = 0$$

$$f_{min} = f(0) = -1$$

Question No: 8

Analysis

A box contains three white & four red balls. Two balls are drawn randomly in sequence. If the first draw resulted in a red ball, the probability of getting a second red ball in the next draw is

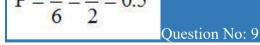
(A) 0.33 (B) 0.50

(C) 0.67 (D) 4.75

Not Attempted -- Correct Answer

: B Solution :

 $P = \frac{3}{6} = \frac{1}{2} = 0.5$



Analysis

If $f(x) = e^{|x|}$ then at x = 0, the function f(x) is

(A) continuous and differentiable(B) continuous but not differentiable(C) neither continuous nor differentiable

(D) not continuous but differentiable

Not Attempted -- Correct Answer : B

Solution :

$f(x) = e^{ x } = e^x \ , \ x > 0$		
$= e^{-x}, x < 0$		
= 1 , $x = 0$		
$f'(x) = e^x , \qquad x > 0$		
$=-e^{-x}$, $x < 0$		
= 0 , $x = 0$		
$f'(0^-) = f'(0^+)$		
\therefore f(x) is continuous but not different	able Question No: 10	

Analysis

Number of ways we can distribute 5 red balls, 5 white balls and 5 blue

balls into 3 different boxes is _____

Not Attempted -- Correct Answer : 9261 & Valid Answer Range : 9261,9261 Solution :

Number of ways we can distribute 5 red balls into 3 numbered boxes

$$= C(3-1+5,5)$$

= 21

Similarly we can distribute 5 white balls in 21 ways and 5 blue balls in 21

ways.

By product rule, required number of ways = (21)(21)(21)

= 9261

Section Title : Two Marks Questions

Total Questions: 20 Max Marks : 2 -ve Marks :0.66

Question No: 11

Analysis

The solution of the system of linear equations

$$x + y + z = 12$$
, $2x + 3y + 3z = 33$, $x - 2y + z = 0$ is

(A) x = 3, y = 4, z = 5 (B) x = -3, y = -4, z = -5

(C) x = 7, y = 0, z = 5 (D) x = 0, y = 6, z = 6

Not Attempted -- Correct Answer : A

Solution :

Consider $[A/B] = \begin{bmatrix} 1 & 1 & 1 & 12 \\ 2 & 3 & 3 & 33 \\ 1 & -2 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 12 \\ 0 & 1 & 1 & 9 \\ 0 & -3 & 0 & -12 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 12 \\ 0 & 1 & 1 & 9 \\ 0 & 0 & 3 & 15 \end{bmatrix}$	
From the above Echelon form of augmented matrix [A/B], rewrite the	
system of linear equations as	
x + y + z = 12	
y + z = 9	
3z = 15	
\Rightarrow z = 5	
y = 9 - z = 9 - 5 = 4	
x = 12 - y - z = 12 - 4 - 5 = 3	
\therefore The solution of the system is x = 3, y = 4, z = 5	Question No: 12

If $\lambda^3 - P\lambda^2 + Q\lambda - R = 0$ is the characteristic equation of matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ then } \mathbf{Q} = _$$

Not Attempted -- Correct Answer : -1 & Valid Answer Range :-1,-1 Solution :

If λ^3 – $P\lambda^2$ + $Q\lambda$ – R = 0 is a characteristic equation of the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \end{bmatrix}$ then the coefficient of λ is the sum of the cofactors of 0 0 1

diagonal elements of matrix A.

Now
$$Q = (-1)^{1+1} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + (-1)^{2+2} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$$

$$\Rightarrow Q = 1 + 1 - 3$$

$$\therefore Q = -1$$

Analysis

If the system of equations x + 2y + 3z = 0, 3x - 2y - z = 0, $\lambda x + 4y + 5z = 0$

Question No: 13

has a nontrivial solution then $\lambda =$

Not Attempted -- Correct Answer : 0 & Valid Answer Range :0,0 Solution :

If the determinant of coefficient matrix A of the given system of linear equations is singular then the system of linear equations has a non-trivial	
solution.	
Consider $ A = 0$	
$\Rightarrow \begin{vmatrix} 1 & 2 & 3 \\ 3 & -2 & -1 \\ \lambda & 4 & 5 \end{vmatrix} = 0$	
$\Rightarrow (-10 + 4) - 2 (15 + \lambda) + 3 (12 + 2\lambda) = 0$	
$\therefore \lambda = 0$ Analysis	Question No: 14

If ex, xex, x²ex are the linearly independent solutions of a differential equation then the differential equation is

(A)
$$y^{111} - 3y^{11} + 3y^1 - y = 0$$
 (B) $y^{111} + 3y^{11} - 3y^1 + y = 0$

(C)
$$y^{111} - 3y^{11} - 3y^1 - y = 0$$
 (D) $y^{111} + 3y^{11} - 3y^1 - y = 0$

Not Attempted -- Correct Answer

: A

Solution :

Given that ex, xex & x²ex are linearly independent solutions of a differential equation \Rightarrow y = c₁ e^x + c₂ xe^x + c₃ x²e^x is general solutions \Rightarrow y = (c₁ + c₂ x + c₃x²)e^x \Rightarrow m₁ = 1, m₂ = 1 & m₃ = 1 are the roots of the auxiliary equation of a required differential equation. If m₁, m₂, m₃ are the roots of the auxiliary equation of the differential equation then the different equation is given by $y^{111} - (m_1 + m_2 + m_3) y^{11} + (m_1m_2 + m_1m_3 + m_2m_3) y^1 - (m_1m_2m_3)y = 0$... The required differential equation is $y^{111} - [(1) + (1) + (1)] y^{11} + [(1) (1) + (1) (1) + (1) (1)]y^1 - [(1) (1) (1)]$ Hence option (A) is correct. Question No: 15

The integrating factor of the differential equation $\sin(x)\frac{dy}{dx} + 2y = \tan^3\left(\frac{x}{2}\right)$

(A)
$$\tan\left(\frac{x}{2}\right)$$
 (B) $\tan^2\left(\frac{x}{2}\right)$

$$(C) \tan(x)$$

is_____

(D) sin (x)

: B Solution :

Given
$$\sin(x)\frac{dy}{dx} + 2y = \tan^3\left(\frac{x}{2}\right)$$

$$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} + \left(\frac{2}{\sin(x)}\right) y = \frac{\tan^3\left(\frac{x}{2}\right)}{\sin(x)}$$

The above differential equation is a linear D.E

Now the integrating factor is given by

 $I.F = e^{\int \frac{2}{\sin(x)} dx}$ $\Longrightarrow I.F = e^{\int 2. \operatorname{cosec}(x) dx = e^{2 \log \left(\tan \left(\frac{x}{2} \right) \right)}}$ $\Longrightarrow I.F = e^{log\left(tan^2\left(\frac{x}{2}\right)\right)}$ \therefore I.F = tan² $\left(\frac{x}{2}\right)$

Question No: 16

Analysis

Which of the following is the solution of the partial differential equation

 $\frac{\partial \mathbf{u}}{\partial \mathbf{x}} = \frac{2\partial \mathbf{u}}{\partial \mathbf{t}} + \mathbf{u}?$

Not Attempted -- Correct Answer



Solution :

Not Attempted -- Correct Answer : A

Given $u_x = 2u_t + u$ (1)	
Here by verification we choose the answer	
$\mathbf{u} = 6\mathbf{e}^{-3\mathbf{x}-2\mathbf{t}}$	
$\Rightarrow u_x = 6e^{-3x-2t} \frac{\partial}{\partial x} (-3x-2t) = -18e^{-3x-2t}$	
And $u_t = 6e^{-3x-2t} \frac{\partial}{\partial x} (-3x-2t) = -12e^{-3x-2t}$	
L.H.S of (1) = $u_x = -18e^{-3x-2t}$	
R.H.S of (1) = $2u_t + u = 2(-12 e^{-3x-2t}) + 6 e^{-3x-2t} = -18e^{-3x-2t}$	
Here L.H.S of $(1) = R.H.S$ of (1)	
.:. Option (a) is correct which is the solution of the given differential	
equation.	Question No: 17

The type of partial differential equation $3\frac{\partial^2 u}{\partial x^2} + 6\frac{\partial^2 u}{\partial x \partial y} + 2\frac{\partial^2 u}{\partial y^2} = 0$ is

(B) hyperbolic

(C) elliptic

(D) non-linear

Not Attempted -- Correct Answer : B

Solution :

Comparing the given partial differential equation with general equation $A \frac{\partial^2 u}{\partial X^2} + B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} + D \frac{\partial u}{\partial x} + E \frac{\partial u}{\partial y} + Fu = Q$, we get A = 3, B = 6 and C = 2Now $B^2 - 4AC = (6)^2 - 4(3) (2) = 36 - 24 = 12 > 0$ If $B^2 - 4AC > 0$ then the partial differential equation is said to be hyperbolic type. \therefore Option (B) is correct. Analysis

C

If $\overline{f} = (y - x^2 + y^2)\overline{i} + (2xy + x)\overline{j}$ then the value of $\int \overline{f} d\overline{r}$ along the curve

 $x^2 + y^2 = 4$ is _____

Not Attempted -- Correct Answer : 0 & Valid Answer Range : 0,0 Solution :

Given curve is a closed curve so by applying Green's theorem we
evaluate line integral
$$\int_{c} (Mdx + Ndy) = \iint_{R} \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dxdy$$
$$Now \oint_{c} \bar{f}.d\bar{r} = \oint_{c} [(y - x^{2} + y^{2})dx + (2xy + x)dy]$$
$$= \iint_{R} \left(\frac{\partial}{\partial x} (2xy + x) - \frac{\partial}{\partial y} (y - x^{2} + y^{2}) \right) dxdy$$
$$= \iint_{R} [2y + 1) - (1 + 2y)] dxdy$$
$$= \iint_{R} 0 dxdy$$
$$\therefore \oint_{c} \bar{f}.d\bar{r} = 0$$
Question No: 19

The solution of $2xyy^1 = 1+y^2$, y(2) = 3 is

(A) $y^2 = 5x-1$ (B) y = 5x + 1

(C)
$$y = 5x$$
 (D) $y = 5x-1$

Solution :

$$2xy \frac{dy}{dx} = y^{2} + 1$$

$$\frac{2ydy}{1+y^{2}} = \frac{dx}{x}$$

$$\int \frac{2y}{1+y^{2}} dy = \int \frac{dx}{x}$$

$$\Rightarrow \ln (1+y^{2}) = \ln x + \ln c = \ln cx$$

$$\Rightarrow 1+y^{2} = Cx$$

$$\Rightarrow y^{2} = Cx - 1 \qquad \rightarrow (1)$$

$$y(2) = 3$$

$$9 = 2C - 1$$

⇒ C = 5
∴ The solution is
$$y^2 = 5x-1$$

Question No: 20

Analysis

The value of the line integral.

$\oint_{c} (\sin x \, dx + y^2 dy - dz) \text{ where C is the circle } x^2 + y^2 = 16 \text{ and } z = 2 \text{ is } ___$

Not Attempted -- Correct Answer : 0 & Valid Answer Range :0,0 Solution :

Let $\vec{F} = \sin x \vec{i} + y^2 \vec{j} + y^2 \vec{j} - \vec{K}$	
$\operatorname{curl} \vec{F} = \begin{vmatrix} i & j & \vec{K} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \sin x & y^2 & -1 \end{vmatrix} = 0$	
By Stoke's theorem	
$\iint_{s} \operatorname{Curl} \vec{F}.\hat{n} ds = 0$	Question No: 21

If A is a 3×3 matrix and det (A) = 2,

then the value of the determinant det {(adj (adj(adj A-1)))} is

(A) $\frac{1}{512}$	(D) 1
	(B) $\frac{1}{1024}$

(C)
$$\frac{1}{128}$$
 (D) $\frac{1}{25}$



Analysis

Two marbles are drawn in succession from a box containing 10 red, 30 white, 20 blue and 15 orange marbles; replacement being made after each draw find the probability that both are white

(D) $\frac{6}{25}$

(A)
$$\frac{3}{25}$$
 (B) $\frac{4}{25}$

(C) $\frac{1}{5}$

: B Solution :

Not Attempted -- Correct Answer

Total marbles	
= 10+30+20+15 = 75	
P[both are white]	
= P[first is white and second is white]	
$=\frac{30}{75}\times\frac{30}{75}=\frac{4}{25}$	
- crane crane." ascens."	Question No: 23

A fair die is rolled. The probability that first time 3 occurs at the even throw, is

(A)
$$\frac{1}{6}$$
 (B) $\frac{5}{11}$

(C)
$$\frac{6}{11}$$
 (D) $\frac{5}{36}$

Solution :

Let p be probability of getting '3' on a die

$$\Rightarrow p = \frac{1}{6} \& q = \frac{5}{6}$$

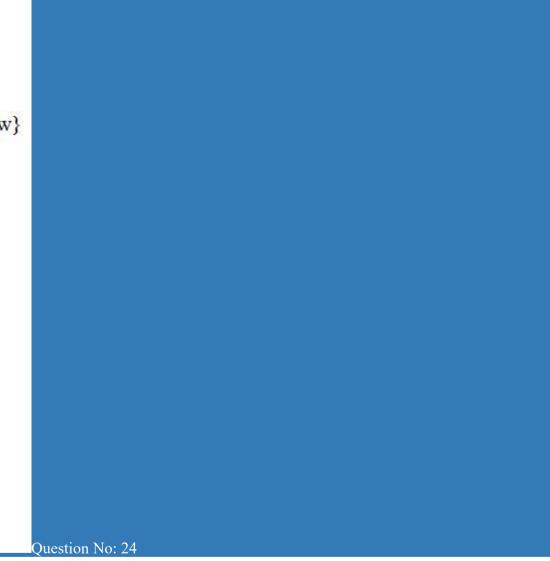
: p{first time 3 occurs at the even throw}

 $= qp + qqqp + qqqqp + \dots$

11

$$= qp \{1+q^{2}+q^{4}+\dots\}$$
$$= qp \left\{\frac{1}{1-q^{2}}\right\}$$
$$= \frac{5}{6} \times \frac{1}{6} \left\{\frac{1}{1-\frac{25}{36}}\right\}$$
$$= \frac{5}{36} \times \frac{36}{11}$$

Not Attempted -- Correct Answer : B



Analysis

To find a root of $f(x) = x + \sqrt{x} - 3 = 0$ using Newton-Raphson method, if the starting value is 2 for the iteration then the next iteration value is



(C) 0.125

(D) 3.572

Not Attempted -- Correct Answer

: A Solution :

By taking
$$f(x) = x + \sqrt{x} - 3$$
 $x_0 = 2$
 $f'(x) = 1 + \frac{1}{2\sqrt{x}}$
 $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1.694$
Question No: 25

Consider $\frac{dy}{dx} = x + y$ with y(0) = 0 using Euler's method with step size of

0.1 the value of y(0.3) is

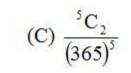
(A) 0.01 (B) 0.031 (C) 0.0631 (D) 0.1 Not Attempted -- Correct Answer : B Solution : In Euler's method we have $y_{n+1} = y_n + nf(x_n, y_n)$ $y_{(0,1)} = y_0 + nf(x_n, y_n)$ = 0 + 0.1(0 + 0) = 0 $y_{(0,2)} = y_{0,1} + nf(x_{0,1}, y_{0,1})$ = 0 + 0.1[0.1 + 0] = 0.01 $y_{(0,3)} = y_{0,2} + nf(x_{0,2}, y_{0,2})$ = 0.01 + 0.1[0.2 + 0.01] = 0.031

Analysis

If there are 5 people in a row, what is the probability that no two of them have same birth day

Question No: 26

(A)
$$\frac{1}{(365)^5}$$
 (B) $\frac{2}{(365)^5}$



(D) $\left(1 - \frac{1}{365}\right) \left(1 - \frac{2}{365}\right) \left(1 - \frac{3}{365}\right) \left(1 - \frac{4}{365}\right)$

Not Attempted -- Correct Answer

: D Solution :

Since birthday of any person can fall on any one of 365 days.

So, total numbers of cases $= 365^5$

If the birthday of all the five persons fall on different days, then the number of favorable cases are (365)(365-1)(365-2)(365-3)(365-4) because in this case the birthday of first person can fall on any one of 365 days, the birthday of second person can fall on the remaining 364 days and so on.

Hence, the probability p, that birthday of all the 5 people are different or the probability that no two of them have the same birthday is given by

$$P = \frac{(365)(365-1)(365-2)(365-3)(365-4)}{(365)^5}$$
$$P = \left(1 - \frac{1}{365}\right) \left(1 - \frac{2}{365}\right) \left(1 - \frac{3}{365}\right) \left(1 - \frac{4}{365}\right)$$

Analysis

The value of $\int_{c} \frac{e^{z}}{(z^{2}+1)} dz$ where c is the circle |z-i| = 4 is

- (A) $2\pi \sinh(i)$ (B) $2\pi \cosh(i)$
- (C) $2\pi \sin(i)$ (D) $2\pi \cos(i)$

Solution :

z = i, -i are singular points lying inside the circle |z - i| = 4

$$\frac{1}{z^{2}+1} = \frac{1}{(z+i)(z-i)} = \frac{1}{2i} \left\{ \frac{1}{z-i} - \frac{1}{z+i} \right\}$$
$$\Rightarrow \frac{e^{z}}{(z^{2}+1)} = \frac{1}{2i} \left\{ \frac{e^{z}}{z-i} - \frac{e^{z}}{z+i} \right\}$$

Integrating both sides w.r.t z, we get

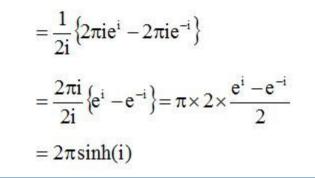
$$\int_{c} \frac{e^{z}}{z^{2}+1} dz = \frac{1}{2i} \left\{ \int_{c} \frac{e^{z}}{z-i} dz - \int_{c} \frac{e^{z}}{z+i} dz \right\}$$

Not Attempted -- Correct Answer : A

Ouestion No: 27

 $=\frac{1}{2i}\{2\pi i f(i) - 2\pi i f(-i)\}$

(by Cauchy's integral formula)

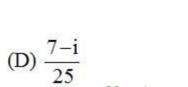




The residue of
$$f(z) = \frac{(z^2 - 2z)}{(z+1)^2(z^2+4)}$$
 at $z = -2i$ is

(A)
$$\frac{7}{25}$$
 (B) $\frac{25-i}{7}$

(C)
$$\frac{i-7}{25}$$



Not Attempted -- Correct Answer : D



$$f(z) = \frac{(z^2 - 2z)}{(z+1)^2(z+2i)(z-2i)}$$

$$z = -2i \text{ is a simple pole}$$
The residue of f(z) at z = -2i is
$$= \frac{(-2i)^2 - 2(-2i)}{(-2i+1)^2(-2i-2i)}$$

$$= \frac{-4+4i}{(-4+1-4i)(-4i)}$$

$$= \frac{-1+i}{i(3+4i)} = \frac{-1+i}{3i-4}$$

$$= \frac{(i-1)(3i+4)}{(3i+4)(3i-4)} = \frac{-7+i}{-25}$$

$$= \frac{7-i}{25}$$

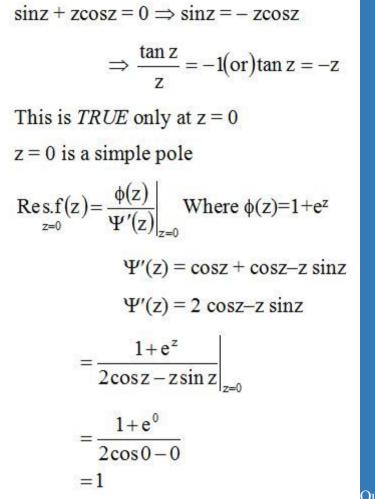
Analysis

The residue of $f(z) = \frac{(1+e^z)}{(\sin z + z \cos z)}$ at z = 0 is

- (A) 0 (B) 3
- (C) 2 (D) 1 Not Attempted -- Correct Answer : D

Solution :

i is i) 2i) i) $= \frac{-7+i}{-25}$ Question No: 29





Question No: 30

The mean value 'c' of cauchy's theorem for the functions $f(x) = \frac{1}{x}$ and

$$g(x) = \frac{1}{x^2}$$
 in the interval [2, 3] is

(A) 2.4 (B) 2.5

(C) 2.6 (D) 2.8

Not Attempted -- Correct Answer

: A Solution :

The conditions of cauchy's theorem hold good for f(x) and g(x). By

cauchy's theorem, there exists a value c such that

$$\frac{f'(c)}{g'(c)} = \frac{f(3) - f(2)}{3 - 2}$$
$$\Rightarrow c = 2.4$$