

Question No: 1

Analysis

If the matrix A of order 5×5 has two non-zero eigen values then the rank of a matrix A

(A) 2

(B) 3

(C) 4

(D) 5

Not Attempted -- Correct Answer : A

Solution :

The rank of the matrix $A_{n \times n}$ is equal to the number of non-zero eigen values

\therefore Option (a) is correct.

Question No: 2

Analysis

Which of the following is the 3rd order and 2nd degree differential equation?

(A) $\left(\frac{d^3y}{dx^3}\right)^4 + \frac{dy}{dx} + \sin(y) = 0$

(B) $\frac{\partial^2 u}{\partial x^2} + \left(\frac{\partial^3 U}{\partial x \partial y^2}\right)^2 + u = 0$

(C) $\left(\frac{\partial^3 u}{\partial x^3}\right)^2 + \left(\frac{\partial^3 u}{\partial y^3}\right)^5 + \frac{du}{dy} = 4$

(D) $\frac{dy^4}{dx^4} + \frac{dy}{dx} + \cos(x) = 0$

Not Attempted -- Correct Answer

: B

Solution :

By the definition of order and degree of the differential equation, option (b) is correct.

Question No: 3

Analysis

If $f(x, y) = x^3 + y^3 - 6xy$ then the point (2, 2) is

(A) a point of minima

(B) a point of maxima

(C) a saddle point

(D) both (A) & (B)

Not Attempted -- Correct Answer : A

Solution :

Given $f(x, y) = x^3 + y^3 - 6xy$

$\Rightarrow P = f_x = 3x^2 - 6y, q = f_y = 3y^2 - 6x$

And $r = f_{xx} = 6x, S = f_{xy} = -6, t = f_{yy} = 6y$

At $(2, 2)$: $r = 12, s = -6, t = 12$

Now $rt - s^2 = (12)(12) - (-6)^2 = 144 - 36 > 0$

But $r = 12 > 0$

$\therefore (2, 2)$ is a point of minima

Question No: 4

Analysis

If $u = \log\left(\frac{x^4 + y^4}{x + y}\right)$ then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \text{_____}$.

(A) 3

(B) 0

(C) -34

(D) 4

Not Attempted -- Correct Answer : A

Solution :

The given function $u(x, y)$ is not a homogeneous function but

$f(u) = e^u = \left(\frac{x^4 + y^4}{x + y}\right)$ is a homogeneous function with degree

$n = 4 - 1 = 3$

By Euler's deduction formula, we have

$$xu_x + yu_y = n \frac{f(u)}{f'(u)}$$

$$\therefore xu_x + yu_y = 3 \frac{e^u}{e^u} = 3$$

Question No: 5

Analysis

Let A be a non-singular matrix of order 3×3 .

If $\det(A) = 3$ then $\det(\text{adj}A^{-1})$ is

(A) $\frac{1}{3}$

(B) $\frac{1}{6}$

(C) $\frac{1}{9}$

(D) $\frac{1}{27}$

Not Attempted -- Correct Answer : C

Solution :

$$\begin{aligned} |\text{adj}(A^{-1})| &= |(A^{-1})|^{n-1} = |A|^{1-n} \\ &= 3^{1-3} \\ &= \frac{1}{9} \end{aligned}$$

Question No: 6

Analysis

The curl of gradient of scalar function

$\phi = x^2 y^2 z^4$ is

(A) -1

(B) 1

(C) 0

(D) 2

Not Attempted -- Correct Answer : C

Solution :

Curl (grad ϕ) = 0 (Vector Identity)

Question No: 7

Analysis

The minimum value taken by the function

$$f(x) = \left[\frac{|x|}{1+|x|} \right] - 1 \text{ is } \underline{\hspace{2cm}}$$

Not Attempted -- Correct Answer : -1 & Valid Answer Range :-1,-1

Solution :

$$f(x) = \frac{|x| - (1+|x|)}{(1+|x|)} = \frac{-1}{1+|x|}$$

$$f'(x) = \frac{1}{(1+|x|)^2} \frac{|x|}{x}$$

$$f'(x) = 0 \Rightarrow x = 0$$

$$f_{\min} = f(0) = -1$$

Question No: 8

Analysis

A box contains three white & four red balls. Two balls are drawn randomly in sequence. If the first draw resulted in a red ball, the probability of getting a second red ball in the next draw is

(A) 0.33

(B) 0.50

(C) 0.67

(D) 4.75

Not Attempted -- Correct Answer

: B

Solution :

$$P = \frac{3}{6} = \frac{1}{2} = 0.5$$

Question No: 9

Analysis

If $f(x) = e^{|x|}$ then at $x = 0$, the function $f(x)$ is

(A) continuous and differentiable

(B) continuous but not differentiable

(C) neither continuous nor differentiable

(D) not continuous but differentiable

Not Attempted -- Correct Answer : B

Solution :

$$f(x) = e^{|x|} = e^x, \quad x > 0$$

$$= e^{-x}, \quad x < 0$$

$$= 1, \quad x = 0$$

$$f'(x) = e^x, \quad x > 0$$

$$= -e^{-x}, \quad x < 0$$

$$= 0, \quad x = 0$$

$$f'(0^-) = f'(0^+)$$

$\therefore f(x)$ is continuous but not differentiable

Question No: 10

Analysis

Number of ways we can distribute 5 red balls, 5 white balls and 5 blue balls into 3 different boxes is _____

Not Attempted -- Correct Answer : 9261 & Valid Answer Range :9261,9261

Solution :

Number of ways we can distribute 5 red balls into 3 numbered boxes

$$= C(3-1+5, 5)$$

$$= 21$$

Similarly we can distribute 5 white balls in 21 ways and 5 blue balls in 21 ways.

By product rule, required number of ways = (21) (21) (21)

$$= 9261$$

Section Title : Two Marks Questions

Total Questions: 20

Max Marks : 2

-ve Marks :0.66

Question No: 11

Analysis

The solution of the system of linear equations

$x + y + z = 12, 2x + 3y + 3z = 33, x - 2y + z = 0$ is

(A) $x = 3, y = 4, z = 5$

(B) $x = -3, y = -4, z = -5$

(C) $x = 7, y = 0, z = 5$

(D) $x = 0, y = 6, z = 6$

Not Attempted -- Correct Answer : A

Solution :

$$\text{Consider } [A/B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 12 \\ 2 & 3 & 3 & 33 \\ 1 & -2 & 1 & 0 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 12 \\ 0 & 1 & 1 & 9 \\ 0 & -3 & 0 & -12 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 12 \\ 0 & 1 & 1 & 9 \\ 0 & 0 & 3 & 15 \end{array} \right]$$

From the above Echelon form of augmented matrix $[A/B]$, rewrite the system of linear equations as

$$x + y + z = 12$$

$$y + z = 9$$

$$3z = 15$$

$$\Rightarrow z = 5$$

$$y = 9 - z = 9 - 5 = 4$$

$$x = 12 - y - z = 12 - 4 - 5 = 3$$

\therefore The solution of the system is $x = 3, y = 4, z = 5$

Question No: 12

Analysis

If $\lambda^3 - P\lambda^2 + Q\lambda - R = 0$ is the characteristic equation of matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ then } Q = \underline{\hspace{2cm}}$$

Not Attempted -- Correct Answer : -1 & Valid Answer Range :-1,-1

Solution :

If $\lambda^3 - P\lambda^2 + Q\lambda - R = 0$ is a characteristic equation of the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ then the coefficient of } \lambda \text{ is the sum of the cofactors of}$$

diagonal elements of matrix A.

$$\text{Now } Q = (-1)^{1+1} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + (-1)^{2+2} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$$

$$\Rightarrow Q = 1 + 1 - 3$$

$$\therefore Q = -1$$

Question No: 13

Analysis

If the system of equations $x + 2y + 3z = 0, 3x - 2y - z = 0, \lambda x + 4y + 5z = 0$

has a nontrivial solution then $\lambda = \underline{\hspace{2cm}}$

Not Attempted -- Correct Answer : 0 & Valid Answer Range :0,0

Solution :

If the determinant of coefficient matrix A of the given system of linear equations is singular then the system of linear equations has a non-trivial solution.

Consider $|A| = 0$

$$\Rightarrow \begin{vmatrix} 1 & 2 & 3 \\ 3 & -2 & -1 \\ \lambda & 4 & 5 \end{vmatrix} = 0$$

$$\Rightarrow (-10 + 4) - 2(15 + \lambda) + 3(12 + 2\lambda) = 0$$

$$\therefore \lambda = 0$$

Question No: 14

Analysis

If e^x , xe^x , x^2e^x are the linearly independent solutions of a differential equation then the differential equation is _____

(A) $y^{111} - 3y^{11} + 3y^1 - y = 0$

(B) $y^{111} + 3y^{11} - 3y^1 + y = 0$

(C) $y^{111} - 3y^{11} - 3y^1 - y = 0$

(D) $y^{111} + 3y^{11} - 3y^1 - y = 0$

Not Attempted -- Correct Answer

: A

Solution :

Given that e^x , xe^x & x^2e^x are linearly independent solutions of a differential equation

$\Rightarrow y = c_1 e^x + c_2 xe^x + c_3 x^2e^x$ is general solutions

$$\Rightarrow y = (c_1 + c_2 x + c_3 x^2)e^x$$

$\Rightarrow m_1 = 1, m_2 = 1$ & $m_3 = 1$ are the roots of the auxiliary equation of a required differential equation.

If m_1, m_2, m_3 are the roots of the auxiliary equation of the differential equation then the differential equation is given by

$$y^{111} - (m_1 + m_2 + m_3) y^{11} + (m_1 m_2 + m_1 m_3 + m_2 m_3) y^1 - (m_1 m_2 m_3) y = 0$$

\therefore The required differential equation is

$$y^{111} - [(1) + (1) + (1)] y^{11} + [(1)(1) + (1)(1) + (1)(1)] y^1 - [(1)(1)(1)]$$

Hence option (A) is correct.

Question No: 15

Analysis

The integrating factor of the differential equation $\sin(x) \frac{dy}{dx} + 2y = \tan^3\left(\frac{x}{2}\right)$ is _____

(A) $\tan\left(\frac{x}{2}\right)$

(B) $\tan^2\left(\frac{x}{2}\right)$

(C) $\tan(x)$

(D) $\sin(x)$

Not Attempted -- Correct Answer

: B

Solution :

Given $\sin(x) \frac{dy}{dx} + 2y = \tan^3\left(\frac{x}{2}\right)$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{2}{\sin(x)}\right)y = \frac{\tan^3\left(\frac{x}{2}\right)}{\sin(x)}$$

The above differential equation is a linear D.E

Now the integrating factor is given by

$$I.F = e^{\int \frac{2}{\sin(x)} dx}$$

$$\Rightarrow I.F = e^{\int 2 \operatorname{cosec}(x) dx} = e^{2 \log\left(\tan\left(\frac{x}{2}\right)\right)}$$

$$\Rightarrow I.F = e^{\log\left(\tan^2\left(\frac{x}{2}\right)\right)}$$

$$\therefore I.F = \tan^2\left(\frac{x}{2}\right)$$

Question No: 16

Analysis

Which of the following is the solution of the partial differential equation

$$\frac{\partial u}{\partial x} = \frac{2\partial u}{\partial t} + u?$$

(A) $u = 6e^{-3x-2t}$

(B) $u = \frac{7}{2}e^{-3x}$

(C) $u = e^{-2t}$

(D) $u = e^{xt}$

Not Attempted -- Correct Answer : A

Solution :

Given $u_x = 2u_t + u$ (1)

Here by verification we choose the answer

$$u = 6e^{-3x-2t}$$

$$\Rightarrow u_x = 6e^{-3x-2t} \frac{\partial}{\partial x}(-3x-2t) = -18e^{-3x-2t}$$

$$\text{And } u_t = 6e^{-3x-2t} \frac{\partial}{\partial t}(-3x-2t) = -12e^{-3x-2t}$$

$$\text{L.H.S of (1)} = u_x = -18e^{-3x-2t}$$

$$\text{R.H.S of (1)} = 2u_t + u = 2(-12e^{-3x-2t}) + 6e^{-3x-2t} = -18e^{-3x-2t}$$

Here L.H.S of (1) = R.H.S of (1)

\therefore Option (a) is correct which is the solution of the given differential equation.

Question No: 17

Analysis

The type of partial differential equation $3\frac{\partial^2 u}{\partial x^2} + 6\frac{\partial^2 u}{\partial x \partial y} + 2\frac{\partial^2 u}{\partial y^2} = 0$ is

(A) parabolic

(B) hyperbolic

(C) elliptic

(D) non-linear

Not Attempted -- Correct Answer : B

Solution :

Comparing the given partial differential equation with general

equation $A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} + Fu = Q$, we get

$$A = 3, B = 6 \text{ and } C = 2$$

$$\text{Now } B^2 - 4AC = (6)^2 - 4(3)(2) = 36 - 24 = 12 > 0$$

If $B^2 - 4AC > 0$ then the partial differential equation is said to be hyperbolic type.

\therefore Option (B) is correct.

Question No: 18

Analysis

If $\vec{f} = (y - x^2 + y^2)\vec{i} + (2xy + x)\vec{j}$ then the value of $\int_c \vec{f} \cdot d\vec{r}$ along the curve

$$x^2 + y^2 = 4 \text{ is } \underline{\hspace{2cm}}$$

Not Attempted -- Correct Answer : 0 & Valid Answer Range : 0,0

Solution :

Given curve is a closed curve so by applying Green's theorem we evaluate line integral

$$\int_c (Mdx + Ndy) = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx dy$$

$$\begin{aligned} \text{Now } \oint_c \vec{f} \cdot d\vec{r} &= \oint_c [(y - x^2 + y^2)dx + (2xy + x)dy] \\ &= \iint_R \left(\frac{\partial}{\partial x} (2xy + x) - \frac{\partial}{\partial y} (y - x^2 + y^2) \right) dx dy \\ &= \iint_R [2y + 1 - (1 + 2y)] dx dy \\ &= \iint_R 0 \, dx dy \end{aligned}$$

$$\therefore \oint_c \vec{f} \cdot d\vec{r} = 0$$

Question No: 19

Analysis

The solution of $2xyy^1 = 1+y^2$, $y(2) = 3$ is

(A) $y^2 = 5x-1$

(B) $y = 5x + 1$

(C) $y = 5x$

(D) $y = 5x-1$

Not Attempted -- Correct Answer : A

Solution :

$$2xy \frac{dy}{dx} = y^2 + 1$$

$$\frac{2ydy}{1+y^2} = \frac{dx}{x}$$

$$\int \frac{2y}{1+y^2} dy = \int \frac{dx}{x}$$

$$\Rightarrow \ln(1+y^2) = \ln x + \ln c = \ln cx$$

$$\Rightarrow 1+y^2 = Cx$$

$$\Rightarrow y^2 = Cx-1 \quad \rightarrow (1)$$

$$y(2) = 3$$

$$9 = 2C-1$$

$$\Rightarrow C = 5$$

$$\therefore \text{The solution is } y^2 = 5x-1$$

Question No: 20

Analysis

The value of the line integral.

$$\oint_c (\sin x \, dx + y^2 dy - dz) \text{ where } C \text{ is the circle } x^2+y^2 = 16 \text{ and } z = 2 \text{ is } \underline{\hspace{2cm}}$$

Not Attempted -- Correct Answer : 0 & Valid Answer Range : 0,0

Solution :

Let $\vec{F} = \sin x \vec{i} + y^2 \vec{j} + y^2 \vec{j} - \vec{k}$

$$\text{curl } \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \sin x & y^2 & -1 \end{vmatrix} = 0$$

By Stoke's theorem

$$\iint_S \text{Curl } \vec{F} \cdot \hat{n} ds = 0$$

Question No: 21

Analysis

If A is a 3×3 matrix and $\det(A) = 2$,

then the value of the determinant $\det \{(\text{adj}(\text{adj}(\text{adj } A^{-1})))\}$ is

(A) $\frac{1}{512}$

(B) $\frac{1}{1024}$

(C) $\frac{1}{128}$

(D) $\frac{1}{256}$

Not Attempted -- Correct Answer : D

Solution :

$$|A| = 2$$

$$\begin{aligned} |\text{adj}(\text{adj}(\text{adj } A^{-1}))| &= |(A^{-1})|^{(n-1)^3} \\ &= |A|^{-(n-1)^3} \\ &= 2^{-8} = \frac{1}{256} \end{aligned}$$

Question No: 22

Analysis

Two marbles are drawn in succession from a box containing 10 red, 30 white, 20 blue and 15 orange marbles; replacement being made after each draw find the probability that both are white

(A) $\frac{3}{25}$

(B) $\frac{4}{25}$

(C) $\frac{1}{5}$

(D) $\frac{6}{25}$

Not Attempted -- Correct Answer

: B

Solution :

Total marbles

$$= 10+30+20+15 = 75$$

P[both are white]

= P[first is white and second is white]

$$= \frac{30}{75} \times \frac{30}{75} = \frac{4}{25}$$

Question No: 23

Analysis

A fair die is rolled. The probability that first time 3 occurs at the even throw, is

(A) $\frac{1}{6}$

(B) $\frac{5}{11}$

(C) $\frac{6}{11}$

(D) $\frac{5}{36}$

Not Attempted -- Correct Answer : B

Solution :

Let p be probability of getting '3' on a die

$$\Rightarrow p = \frac{1}{6} \quad \& \quad q = \frac{5}{6}$$

\therefore p {first time 3 occurs at the even throw}

$$= qp + qqqp + qqqqp + \dots$$

$$= qp \{1 + q^2 + q^4 + \dots\}$$

$$= qp \left\{ \frac{1}{1 - q^2} \right\}$$

$$= \frac{5}{6} \times \frac{1}{6} \left\{ \frac{1}{1 - \frac{25}{36}} \right\}$$

$$= \frac{5}{36} \times \frac{36}{11}$$

$$= \frac{5}{11}$$

Question No: 24

Analysis

To find a root of $f(x) = x + \sqrt{x} - 3 = 0$ using Newton-Raphson method, if the starting value is 2 for the iteration then the next iteration value is

(A) 1.694

(B) 2.135

(C) 0.125

(D) 3.572

Not Attempted -- Correct Answer

: A

Solution :

By taking $f(x) = x + \sqrt{x} - 3$ $x_0 = 2$

$$f'(x) = 1 + \frac{1}{2\sqrt{x}}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1.694$$

Question No: 25

Analysis

Consider $\frac{dy}{dx} = x + y$ with $y(0) = 0$ using Euler's method with step size of

0.1 the value of $y(0.3)$ is

(A) 0.01

(B) 0.031

(C) 0.0631

(D) 0.1

Not Attempted -- Correct Answer

: B

Solution :

In Euler's method we have

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{(0.1)} = y_0 + hf(x_0, y_0)$$

$$= 0 + 0.1(0 + 0) = 0$$

$$y_{(0.2)} = y_{0.1} + hf(x_{0.1}, y_{0.1})$$

$$= 0 + 0.1[0.1 + 0] = 0.01$$

$$y_{(0.3)} = y_{0.2} + hf(x_{0.2}, y_{0.2})$$

$$= 0.01 + 0.1[0.2 + 0.01] = 0.031$$

Question No: 26

Analysis

If there are 5 people in a row, what is the probability that no two of them have same birth day

(A) $\frac{1}{(365)^5}$

(B) $\frac{2}{(365)^5}$

(C) $\frac{{}^5C_2}{(365)^5}$

(D) $\left(1 - \frac{1}{365}\right)\left(1 - \frac{2}{365}\right)\left(1 - \frac{3}{365}\right)\left(1 - \frac{4}{365}\right)$

Not Attempted -- Correct Answer

: D

Solution :

Since birthday of any person can fall on any one of 365 days.

So, total numbers of cases = 365^5

If the birthday of all the five persons fall on different days, then the number of favorable cases are $(365)(365-1)(365-2)(365-3)(365-4)$ because in this case the birthday of first person can fall on any one of 365 days, the birthday of second person can fall on the remaining 364 days and so on.

Hence, the probability p , that birthday of all the 5 people are different or the probability that no two of them have the same birthday is given by

$$P = \frac{(365)(365-1)(365-2)(365-3)(365-4)}{(365)^5}$$

$$P = \left(1 - \frac{1}{365}\right) \left(1 - \frac{2}{365}\right) \left(1 - \frac{3}{365}\right) \left(1 - \frac{4}{365}\right)$$

Question No: 27

Analysis

The value of $\int_c \frac{e^z}{(z^2+1)} dz$ where c is the circle $|z-i|=4$ is

(A) $2\pi \sinh(i)$

(B) $2\pi \cosh(i)$

(C) $2\pi \sin(i)$

(D) $2\pi \cos(i)$

Not Attempted -- Correct Answer : A

Solution :

$z = i, -i$ are singular points lying inside the circle $|z-i|=4$

$$\frac{1}{z^2+1} = \frac{1}{(z+i)(z-i)} = \frac{1}{2i} \left\{ \frac{1}{z-i} - \frac{1}{z+i} \right\}$$

$$\Rightarrow \frac{e^z}{(z^2+1)} = \frac{1}{2i} \left\{ \frac{e^z}{z-i} - \frac{e^z}{z+i} \right\}$$

Integrating both sides w.r.t z , we get

$$\int_c \frac{e^z}{z^2+1} dz = \frac{1}{2i} \left\{ \int_c \frac{e^z}{z-i} dz - \int_c \frac{e^z}{z+i} dz \right\}$$

$$= \frac{1}{2i} \{2\pi i f(i) - 2\pi i f(-i)\}$$

(by Cauchy's integral formula)

$$= \frac{1}{2i} \{2\pi i e^i - 2\pi i e^{-i}\}$$

$$= \frac{2\pi i}{2i} \{e^i - e^{-i}\} = \pi \times 2 \times \frac{e^i - e^{-i}}{2}$$

$$= 2\pi \sinh(i)$$

Question No: 28

Analysis

The residue of $f(z) = \frac{(z^2 - 2z)}{(z+1)^2(z^2 + 4)}$ at $z = -2i$ is

(A) $\frac{7}{25}$

(B) $\frac{25-i}{7}$

(C) $\frac{i-7}{25}$

(D) $\frac{7-i}{25}$

Not Attempted -- Correct Answer : D

Solution :

$$f(z) = \frac{(z^2 - 2z)}{(z+1)^2(z+2i)(z-2i)}$$

$z = -2i$ is a simple pole

The residue of $f(z)$ at $z = -2i$ is

$$\begin{aligned} &= \frac{(-2i)^2 - 2(-2i)}{(-2i+1)^2(-2i-2i)} \\ &= \frac{-4+4i}{(-4+1-4i)(-4i)} \\ &= \frac{-1+i}{i(3+4i)} = \frac{-1+i}{3i-4} \\ &= \frac{(i-1)(3i+4)}{(3i+4)(3i-4)} = \frac{-7+i}{-25} \\ &= \frac{7-i}{25} \end{aligned}$$

Question No: 29

Analysis

The residue of $f(z) = \frac{(1+e^z)}{(\sin z + z \cos z)}$ at $z = 0$ is

(A) 0

(B) 3

(C) 2

(D) 1

Not Attempted -- Correct Answer : D

Solution :

$$\sin z + z \cos z = 0 \Rightarrow \sin z = -z \cos z$$

$$\Rightarrow \frac{\tan z}{z} = -1 \text{ (or) } \tan z = -z$$

This is *TRUE* only at $z = 0$

$z = 0$ is a simple pole

$$\text{Res.} f(z) = \frac{\phi(z)}{\Psi'(z)} \Big|_{z=0} \quad \text{Where } \phi(z) = 1 + e^z$$

$$\Psi'(z) = \cos z + \cos z - z \sin z$$

$$\Psi'(z) = 2 \cos z - z \sin z$$

$$= \frac{1 + e^z}{2 \cos z - z \sin z} \Big|_{z=0}$$

$$= \frac{1 + e^0}{2 \cos 0 - 0}$$

$$= 1$$

Question No: 30

Analysis

The mean value 'c' of cauchy's theorem for the functions $f(x) = \frac{1}{x}$ and

$g(x) = \frac{1}{x^2}$ in the interval $[2, 3]$ is

(A) 2.4

(B) 2.5

(C) 2.6

(D) 2.8

Not Attempted -- Correct Answer

: A

Solution :

The conditions of cauchy's theorem hold good for $f(x)$ and $g(x)$. By cauchy's theorem, there exists a value c such that

$$\frac{f'(c)}{g'(c)} = \frac{f(3) - f(2)}{3 - 2}$$

$$\Rightarrow c = 2.4$$